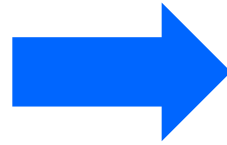
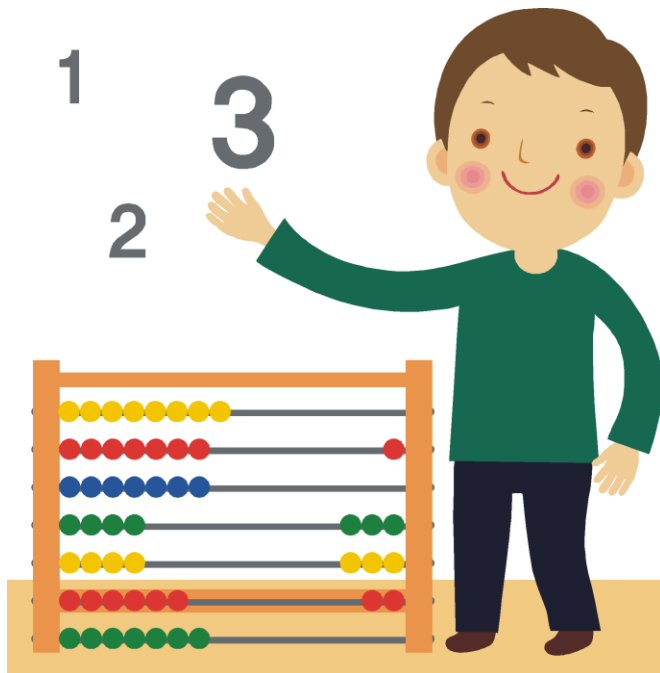


# What factors comprise “early numeracy” and why does no one agree?

**Declan Devlin, Korbinian Moeller & Francesco Sella**

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Loughborough University

# Introduction



**Multi-  
Dimensional :)**

# What is early numeracy?

Measurement

Patterning

Magnitude comparison

Basic arithmetic

Geometry

Counting skills

Graph interpretation

Order processing

Numberline estimation

Strategy selection

Place value understanding

# Who cares???

- Assessment
- Screening for risk of MLD
- Designing early math curricula and targeted interventions
- Research/measurement

# Factor analysis

Factor analysis can be used to extract patterns from a correlation matrix:

|               | Counting up | Estimation | Counting down | Skip counting | Comparison |
|---------------|-------------|------------|---------------|---------------|------------|
| Counting up   | 1           | -          | -             | -             | -          |
| Estimation    | .22         | 1          | -             | -             | -          |
| Counting down | .78         | .13        | 1             | -             | -          |
| Skip counting | .84         | .18        | .87           | 1             | -          |
| Comparison    | .30         | .88        | .24           | .11           | 1          |

1. Counting skills
2. Mapping skills

Milburn et al. (2019)

Four-factor model

1. Number and operation
2. Measurement
3. Geometry
4. Patterning

Hirsch et al. (2018)

Five-factor model

1. Patterning
2. Seriation
3. Non-symbolic comparison

Braeuning et al. (2020)

Four-factor

1. Patterning/Geometry
2. Number sense
3. Arithmetic
4. Data-analysis/statistics

N = 849

Age = 5.01 years

Cirino (2011)

Five-factor model

1. Non-symbolic comparison
2. Symbolic comparison
3. Symbolic labelling/sequencing
4. Rote counting
5. Counting knowledge

N = 278

Age = 6.13 years

**What the f\*ck!?**

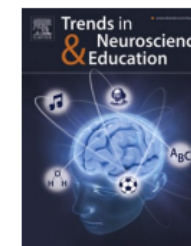


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Review article

### The structure of early numeracy: Evidence from multi-factorial models

Declan Devlin<sup>a,\*</sup>, Korbinian Moeller<sup>a,b,c,d</sup>, Francesco Sella<sup>a</sup>

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#### ARTICLE INFO

##### Keywords:

Early numeracy  
Counting  
Number relations  
Arithmetic  
Factor analysis  
Number development

#### ABSTRACT

**Background:** Despite the relevance of basic numerical skills to later mathematical outcomes, there remains little consensus on which skills comprise the construct of early numeracy.

**Procedure:** Here, we evaluate recent studies that investigated this construct using various approaches.

**Main findings:** Findings from these studies indicated broad consensus in considering skills such as counting, number relations, and basic arithmetic, as central aspects of early numeracy. However, both the number and contents of identified factors varied considerably across studies.

**Conclusions:** Although this review substantiated the critical relevance of certain basic numerical skills, it also highlights a need for future research to evaluate the structure of early numeracy in a more systematic and co-ordinated way to increase comparability and coherence across studies.

# Types of models

**Theory-testing models:** testing theoretical distinctions (e.g., symbolic vs. non-symbolic).

**Core skills models:** identifying the most essential skills for numerical development.

**Curriculum skills models:** including skills aligned with key content strands of early mathematics curricula.

# Theory-testing models

# Conceptual vs. procedural

Ribner et al. (2018)

Two-factor model

1. Conceptual skills
2. Procedural skills

N = 849

Age = 5.01 years

*Pre-K*

Ribner et al. (2018)

## Conceptual skills

- 1-1 correspondence
- Arabic numerals
- Ordinal positioning
- Fill in blank: pattern
- Informal addition
- Shape recognition

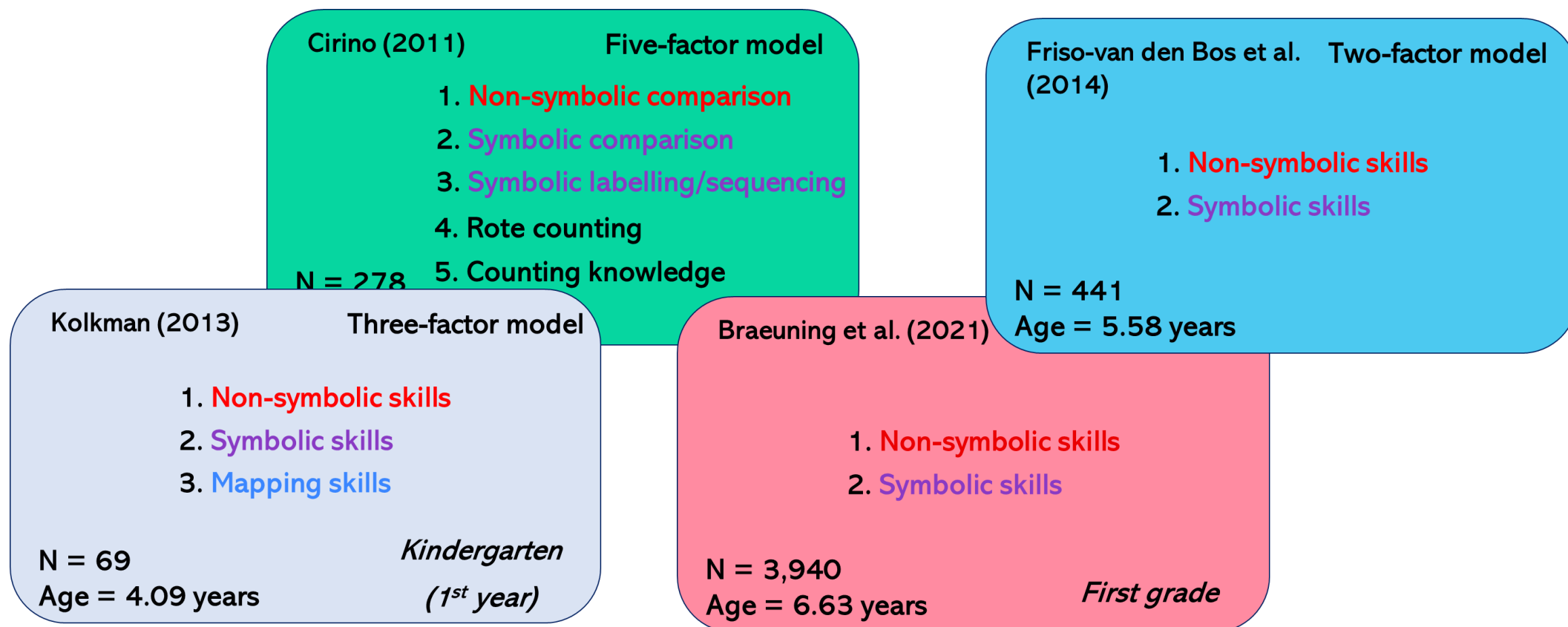
## Procedural skills

- Procedural counting
- Pattern recognition
- Subtraction (with manipulatives)
- Measurement
- Graph interpretation

Procedural skills may be assessed by having children solve arithmetic or counting tasks

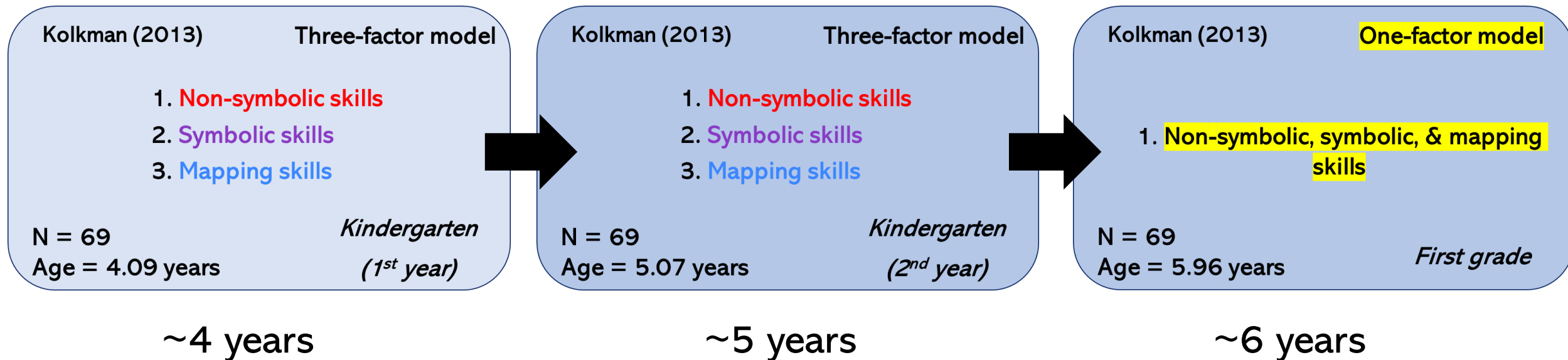
# Symbolic vs. non-symbolic

General Support for a distinction between symbolic and non-symbolic skills:



# Longitudinal evidence

Symbolic/non-symbolic skills may begin as distinct representations of number...



...but may become increasingly integrated over time,  
as we learn to consistently map quantities onto number symbols consistently

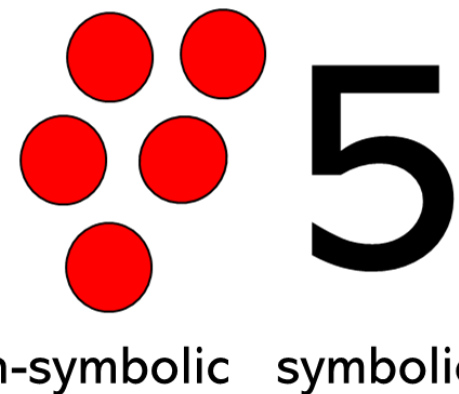
# How we define symbolic/non-symbolic

We typically measure  
and estimation tasks

Braeuning et al. (2021)  
conceptualisation  
standardised mathematics

Braeuning et al. (2021) Two-factor model

1. **Non-symbolic skills**
2. **Symbolic skills**



Sy

lic

N = 3,940

Age = 6.63 years

*First grade*

**Non-symbolic:** numerical information presented in non-symbolic formats (e.g., **sets of dots**).

# Considering theoretical distinctions

Aspects of early numeracy may be differentiated by both their manifest content and the type of processing they require.

Cirino (2011)

Five-factor model

1. **Non-symbolic comparison**
2. **Symbolic comparison**
3. Symbolic labelling/sequencing
4. **Rote counting**
5. **Counting knowledge**

N = 278

Age = 6.13 years

# Core skills models

# Consensus

1. **Numbering**

2. **Relations**

3. **Operations**

The U.S. National Council of Teachers of Mathematics proposed a combined factor of **Numbering** and **Relations**, plus a factor of **Arithmetic operations** as primary aspects of early numeracy (2006).

The U.S. National Research Council proposed **Numbering**, **Relations**, and **Operations** as primary aspects of early numeracy (2009).

# Why only these skills???

Curriculum skills assumed less relevant to later math outcomes, especially MLDs.

For example, arithmetic skills are central to the diagnostic criteria for MLD.

Core skills may be better predictors of later mathematical outcomes than curriculum skills (e.g., [Nguyen et al., 2016](#)).

# Purpura & Lonigan (2013)

## 1. Numbering

Purpura & Lonigan (2013) Three-factor model

## Arithmetic operations

393 preschool

Using the P  
each of the

Compared to  
all possible

1. Numbering

2. Relations

3. Arithmetic operations

N = 393

Age = 4.75 years

9 items for

model and

# Aunio & Räsänen (2016)

Aunio et al. (2019)

Three-factor model

1. Relational skills

2. Counting skills

Lopez-Pedersen et al.  
(2020)

Three-factor model

Hellstrand et al. (2020)

Four-factor model

1. Symbolic & non-symbolic number knowledge

2. Understanding mathematical relations

3. Counting skills

4. Basic arithmetic skills

N = 1139

Age = 6-8 years

N = 44

Age = 6-8 years

Relational skills

Counting skills

Arithmetic skills

# Comparing these two models

Purpura & Lonigan (2013)

1. Numbering

2. Relations

3. Arithmetic operation

Counting skills +  
Nonsymbolic  
estimation

Aunio &

Symbolic/nonsymbolic  
comparison/estimation

1. Symbolic/non-symbolic number sense

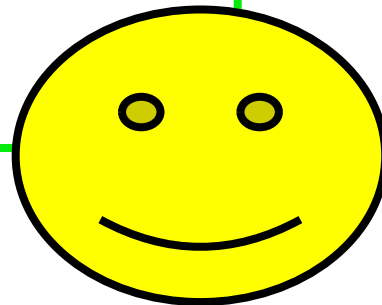
2. Mathematical relations

3. Arithmetic

4. Counting skills

Counting skills

Numeral  
identification



# Curriculum skills models

# Proposed skills

**Number & Operations**

**Measurement**

**Geometry**

**Patterning**

**Data analysis/statistics**

The NRC (2009) and NCTM (2006) recommended **Number & Operations**, **Measurement**, and **Geometry** as key content strands of early maths education.

Milburn et al. (2019) highlight these skills plus **Patterning** are included in various international kindergarten curricula.

# Proposed models

Hirsch et al. (2018)

Five-factor model

1. **Patter**
2. **Ser**
3. **Nor**
4. **Cou**
5. **Syn**

N = 1728

Age = 5.83 years

Milburn et al. (2019)

Four-factor model

1. **Num**
2. **Mea**
3. **Geor**
4. **Patte**

N = 1630

Age = 4.46 years

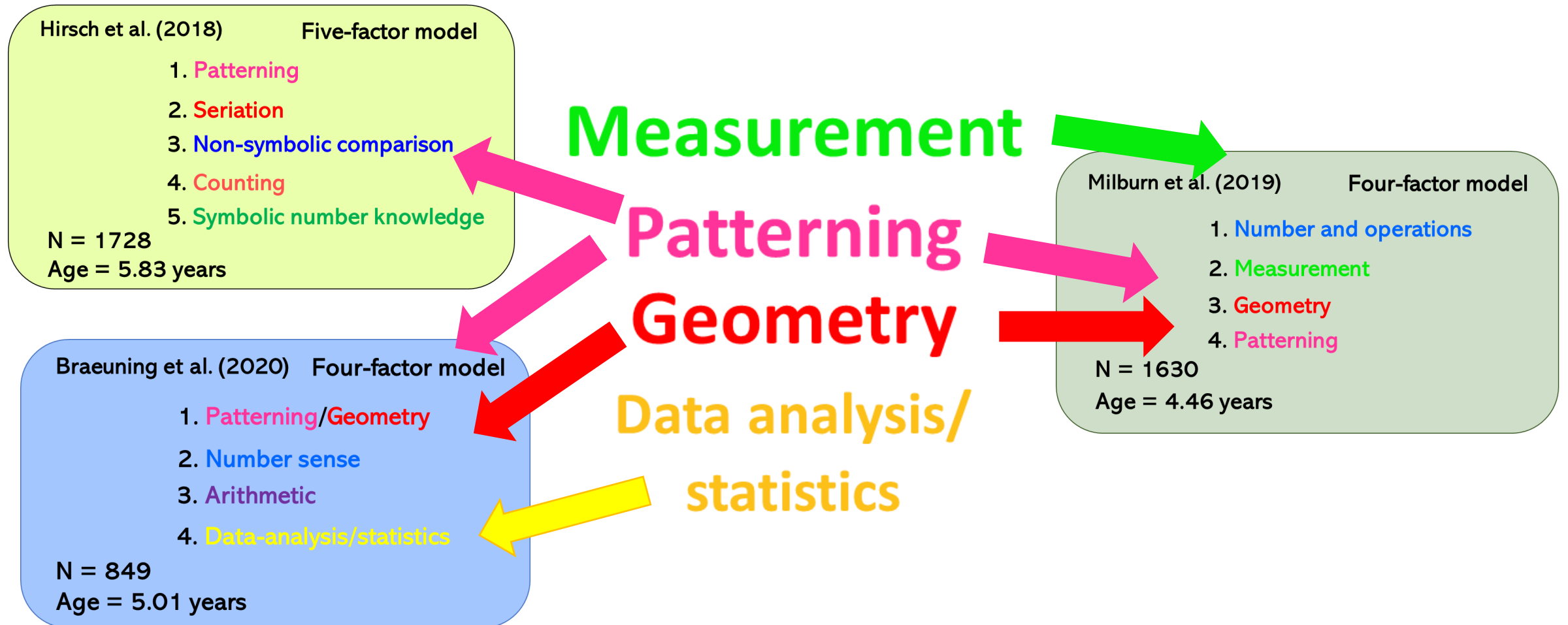
Braeuning et al. (2020) Four-factor model

1. **Patterning/Geometry**
2. **Number sense**
3. **Arithmetic**
4. **Data-analysis/statistics**

N = 849

Age = 5.01 years

# Comparison across models



# Discussion

# Curriculum skills as a predictor of mathematics?

There is only limited data for how well models incorporating curriculum skills predict later mathematical outcomes.

Hirsch et al. (2018)

Five-factor model

1. **Patterning**
2. **Seriation**
3. **Non-symbolic comparison**
4. **Counting**
5. **Symbolic number knowledge**

N = 1728

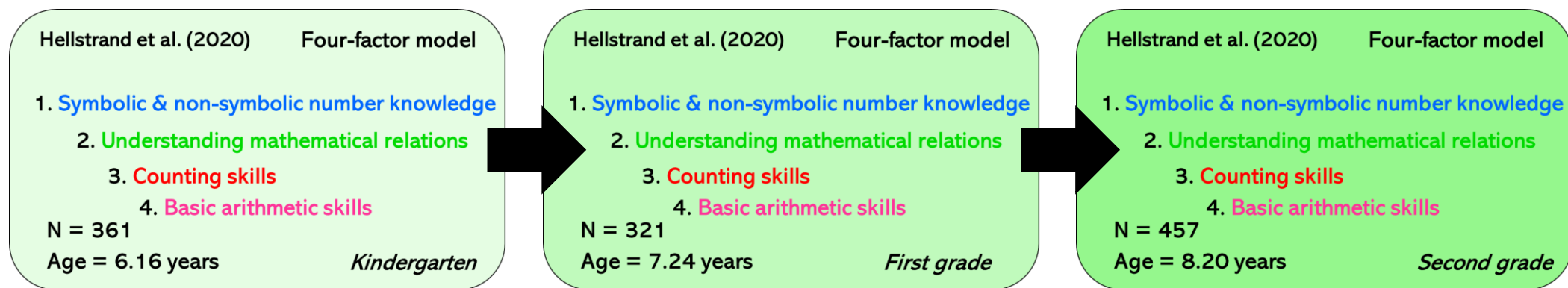
Age = 5.83 years

Hirsch et al. (2018) did longitudinally assess the predictive value of their model, but did not find **patterning** to be uniquely predictive of later mathematics achievement.

**Measurement,**  
**Geometry,**  
**Data analysis???**

# Longitudinal stability

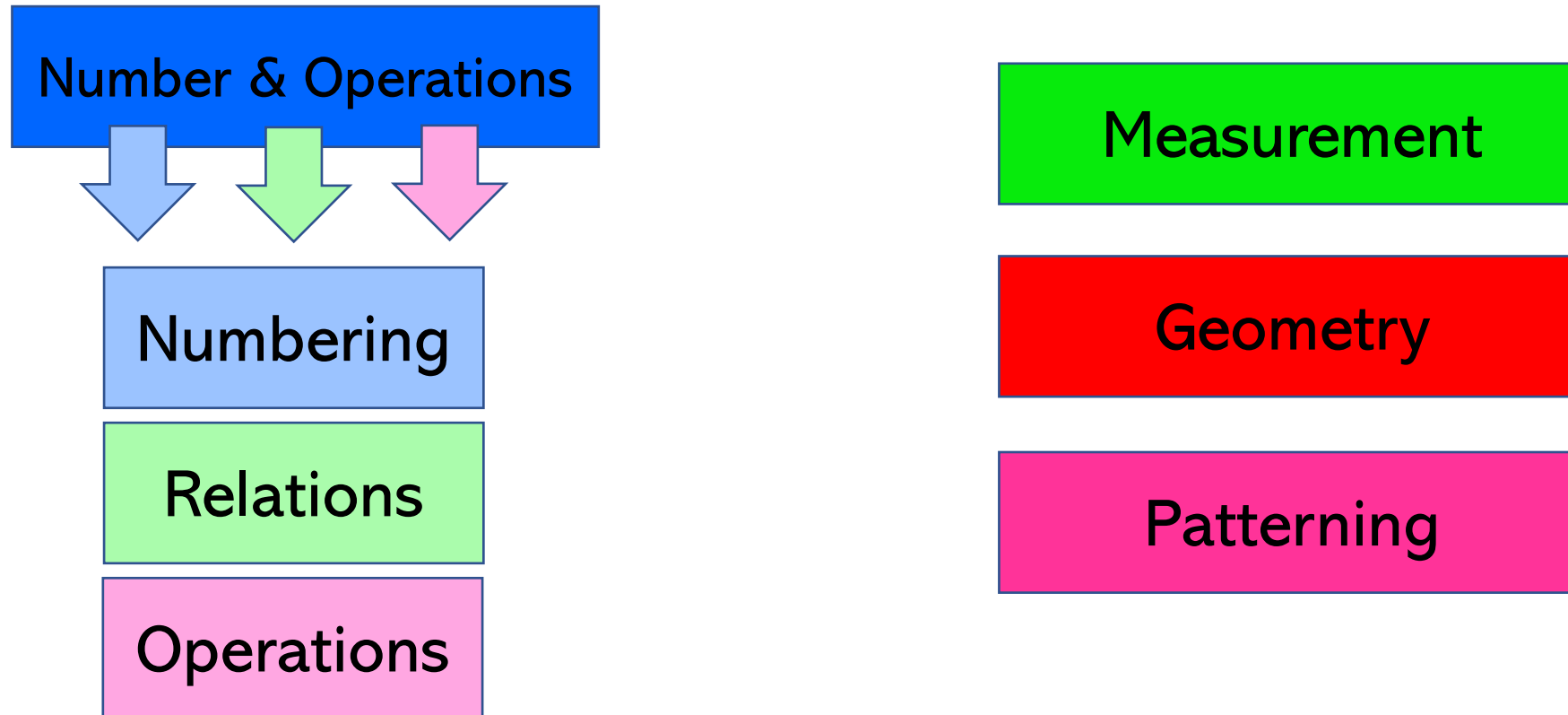
Although many studies have assessed their models at multiple time points, the longitudinal stability of these models over time remains somewhat unclear.



Correlations between skills higher in kindergarten than in second grade, suggesting skills may become more separated overtime.

Similarly, **Ryoo et al. (2015)** observed an increasing number of factors between Prekindergarten and first grade.

# Milburn et al. (2019)



# Proposed framework

## Early Mathematics

### Early Numeracy

Numbering

Relations

Operations

Counting??

Measurement

Geometry

Patterning

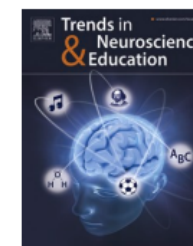
Data/statistics??



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Thanks for  
your  
attention!



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